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ScienceDirect

Energy Procedia 64 (2015) 130 – 139

Energy Procedia

3rd Trondheim Gas Technology Conference, TGTC-3

Simulation and optimization models of steady-state gas transmission networks

Alfredo Bermúdez[a], Julio González-Díaz[b], Francisco J. González-Diéguez[a,*], Ángel M. González-Rueda[b], María P. Fernández de Córdoba[b]

[a]*Dpt. of Applied Mathematics. University of Santiago de Compostela, Campus Vida, Santiago de Compostela 15782, Spain*
[b]*Dpt. of Statistics and Operations Research, University of Santiago de Compostela, Campus Vida, Santiago de Compostela, 15782, Spain*

Abstract

Managing a gas transport network is a complex problem because of the number of possibilities of routing the gas through the pipes. The most important aim in this kind of systems is to fulfill the demand within the pressure bounds, independently of its associated costs. However, in the present work some cost drivers are also taken into account by means of different objective functions in order to manage the network in an efficient way. This work deals with mathematical modeling and optimization of gas transport networks, where a two-stage procedure is proposed. In the first stage, optimization algorithms based on mathematical programming are applied to make some decisions (whether to activate compressor stations, control valves and other control elements) and gives an initial solution to the second stage. This last stage, which is based on control theory techniques, refines the solution to obtain more accurate results. Due to the reduced complexity in each stage, both can be solved within reasonable runtimes for relatively large gas networks. Based on the mathematical methods involved, a software called GANESO™ has been developed.

Peer-review under responsibility of the Scientific Committee of TGTC-3

Keywords: gas network; optimization; mixed integer programming; nonlinear problem; sequential linear programming; control theory

* Corresponding author. Tel.: +34881813122.
E-mail address: franciscojose.gonzalez@usc.es

Peer-review under responsibility of the Scientific Committee of TGTC-3
doi:10.1016/j.egypro.2015.01.016

1. Introduction

A gas network basically consists of a number of controllable elements such as compressor stations and control valves that are connected via pipes. In the pipes of the network, the pressure of flowing gas decreases due to the friction with the walls of the pipes. This pressure loss makes more difficult to guarantee the *security of supply*: to meet the demand at the exit points with gas supplied at the entry points within the pressure bounds. Therefore, compressor stations can be employed to counterbalance the pressure loss, but they consume some proportion of the gas that flows through the pipes. Taking this into account, it is very important to manage the gas transport network efficiently in order to reduce the self-consumption in compressor stations. A mathematical model for this situation along with a series of algorithms has been developed to determine how to route the gas so that the demands are met and certain objective functions are optimized: for example to minimize the cost of the compressors.

This project has been developed as a turn-key project to release a user-friendly computer program for Reganosa LNG. This program is described in Section 5 and used in an example of the optimization of the Spanish gas network in Section 6.

Some of the topics involved in this development are already discussed in the literature. In [9] mixed integer models for the stationary case of gas network are proposed but without operating diagram constraints, feature included in our work, and for networks without cycles, which is not the case of Spanish gas network. Other related works are presented in [10] and [11].

2. Modeling

The modeling can be divided in two parts: the first one is related to how to represent the topology of the network whereas the second one is focused on the mathematical models which reproduce the physical behavior.

2.1. Topology of the network

The gas transport network is modeled as a directed graph **G** = (**N**, **E**), where **N** represents the set of *n* nodes and **E** the set of *e* edges. Thus, each element from **E** is an ordered pair of elements from **N**.

On the one hand, the nodes represent the following network elements: gas supply points; gas consumption points; underground storages; suction points or discharge points in a compressor station; interconnection points among gas pipes; and points where the properties of the gas pipe changes (diameter, roughness, …).

On the other hand, the edges correspond to: structural pipes between nodes; compressors, where each compressor links the suction node and the discharge node by a ratio of pressures of these nodes; flow control valves (FCV), where the mass flow of the pipe is imposed; closed closing valves, where the mass flow is zero; bypass or opened closing valves, where there is no pressure loss; and pressure control valves (PCV), which link the two nodes of the edge by a ratio of pressures of these nodes.

2.2. Mathematical model

The mathematical model can be deduced from the Navier-Stokes equations for compressible flows: conservation of mass, conservation of momentum and conservation of energy (see [4]). In order to complete the model, it is also necessary to apply the following constitutive laws: Newtonian viscous fluid that accounts for the friction , equation of state for real gases and Fourier's law for the heat flow (see [4]).

All these equations lead to a model with three spatial dimensions in which we do some standard simplifications:

- We consider that the temperature is a datum; that is, it is a constant parameter of the model, so we remove the equation of conservation of energy from the model. This is common in practice (see e.g. [8]).
- We integrate the equations in the circular cross-section to obtain a one-dimensional model.
- We adapt this one-dimensional model to get a steady-state model.
- Finally, we approximate the one-dimensional model thanks to the integration between the endpoints of the pipe.

After applying these simplifications, we arrive at a family of equations formed by the conservation of mass at nodes and the pressure loss at edges which reproduces the physical behavior of the gas network in steady-state. Therefore, at this point we must identify the *variables* of the model. Initially, the unknowns of the model could be:

- The *square pressure* [Pa^2] at nodes: $\{u_i: i = 1, \ldots, n\}$. Let us denote by $\mathbf{u}$ the column vector of n components: $\mathbf{u} = (u_1, \ldots, u_n)^t$.
- The *exchanged mass flow rate* [kg/s] with the exterior at nodes; that is, the amount of gas consumed (negative value) or supplied (positive value) at the nodes: $\{c_i: i = 1, \ldots, n\}$. Let us denote by $\mathbf{c}$ the column vector of n components: $\mathbf{c} = (c_1, \ldots, c_n)^t$.
- The *mass flow rate* [kg/s] at edges: $\{q_j: j = 1, \ldots, e\}$. Let us denote by $\mathbf{q}$ the column vector of e components: $\mathbf{q} = (q_1, \ldots, q_e)^t$.

The conservation of mass establishes that, at any node, the sum of the ingoing mass flows must be equal to the sum of outgoing mass flows. Thanks to the incidence matrix of the graph which represents the network, $\mathbf{A}$, the law of conservation of mass can be written in matrix form as

$$\mathbf{A}\mathbf{q} = \mathbf{c}. \tag{1}$$

Since it is a steady-state model, notice that $c_1 + c_2 + \ldots + c_n = 0$.

The pressure loss equation establishes that there exists a head loss lengthwise of a pipe, mainly due to viscous stress (friction with the walls of the pipe), which can be computed with the following function:

$$\begin{aligned} G_j\left(u_{ini(j)}, u_{fin(j)}, q_j\right) = {} & \frac{16 L_j R}{\pi^2 D_j^5}\theta_a Z\left(\sqrt{u_a}, \theta_a\right)\lambda\left(\left|q_j\right|\right)\left|q_j\right| q_j \\ & + \frac{2g}{R\theta_a}\frac{u_a}{Z\left(\sqrt{u_a}, \theta_a\right)}\left(H_{fin(j)} - H_{ini(j)}\right), \end{aligned} \tag{2}$$

where $u_{ini(j)}$ and $u_{fin(j)}$ are, respectively, the square pressure at the beginning and at the end of the pipe, u_a is the *average square pressure*, q_j is mass flow rate of the pipe, θ_a is the *average absolute temperature* [K], $H_{ini(j)}$ and $H_{fin(j)}$ are, respectively, the heights [m] at the beginning and at the end of the pipe, L_j and D_j are, respectively, the length [m] and the diameter [m] of the section of the pipe, g is the gravity acceleration [$\mathrm{m/s}^2$], R is the *gas constant* [J/(kg K)], $\lambda(|q_j|)$ is the *Darcy friction factor* [dimensionless], and $Z(\sqrt{u_a}, \theta_a)$ is the *compressibility factor* [dimensionless] of the gas (see [6]).

Let us recall that the compressibility factor can be computed with the AGA8 (see [2]) empirical equation or the SGERG88 (see [1]) procedure, whereas the Darcy friction factor can be calculated with the Weymouth formula or the Colebrook equation (see [5]) among others.

As we have seen, the edges can represent different types of pipes. Above we have presented the pressure loss equation for structural pipes and below we will define the pressure loss equations for compressors (3), pressure control valves (4); and bypasses or opened closing valves (5).

$$G_j\left(u_{ini(j)}, u_{fin(j)}, q_j\right) = u_{ini(j)}\left(1 - \frac{u_{fin(j)}}{u_{ini(j)}}\right), \quad \text{where} \quad u_{fin(j)} > u_{ini(j)} \tag{3}$$

$$G_j\left(u_{ini(j)}, u_{fin(j)}, q_j\right) = u_{ini(j)}\left(1 - \frac{u_{fin(j)}}{u_{ini(j)}}\right), \quad \text{where} \quad u_{fin(j)} < u_{ini(j)} \tag{4}$$

$$G_j\left(u_{ini(j)}, u_{fin(j)}, q_j\right) = 0. \tag{5}$$

On the other hand, the compressor stations need a source of energy to work, which is generally the gas itself that is in the network. Then, as one of the optimization goals one can consider in our model, we have built another equation to reproduce the self-consumption [kg/s] in the compressor stations as follows:

$$Q = \frac{1}{\xi\, LCV}\frac{\gamma}{\gamma-1} Z\left(\sqrt{u_a}, \theta_a\right) R\theta_{ini}\left(\left(\frac{u_{fin}}{u_{ini}}\right)^{\frac{\gamma-1}{2\gamma}} - 1\right) q, \tag{6}$$

being θ_{ini} the absolute temperature [K] at the entrance of the compressor, LCV [J/kg] the lower calorific value, γ the ratio of specific heats [dimensionless] and ξ a factor of efficiency [dimensionless] involving the isentropic efficiency of the process and the mechanical efficiency of the compressor.

3. Simulation

The simulator of the gas transport network collects all the equations of the model we have just defined in the previous section. In particular, we have built the following nonlinear system, which, in the context of the mathematical systems theory, is the so-called state equation:

$$\begin{pmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{A}^t & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{u} \\ \mathbf{q} \end{pmatrix} - \begin{pmatrix} \mathbf{0} \\ G(\mathbf{u},\mathbf{q}) \end{pmatrix} = \begin{pmatrix} \mathbf{c} \\ \mathbf{0} \end{pmatrix}. \tag{7}$$

Notice that this is a nonlinear system because of the term involving the pressure loss. In particular, this system can be solved with iterative algorithms, e.g. Newton-like algorithms.

It is important to point out that, in the simulator, the two unknowns at each node are, in a sense, complements of each other: one of them can be computed but the other one must be imposed. For example: if the square pressure is imposed at a certain node, the unknown will be the exchanged mass flow rate with the exterior. If the exchanged mass flow rate with the exterior is imposed at a certain node, the unknown will be the square pressure at this node.

Additionally, it is mandatory to impose the pressure at least at one node to guarantee uniqueness of solution.

4. Optimization

In order to optimize the gas transport network, different optimization goals can be considered: minimize the self-consumption in the compressor stations, minimize the boil-off gas in the regasification plants, maximize the exportation from any area of the network to another area or reduce the bottlenecks. For the sake of simplicity, in this paper we will focus on minimizing the self-consumption in the compressor stations, that is, we want to minimize the function defined in (5).

This minimization is achieved by modifying the compression ratio at the compressor stations, decompression ratio at PCVs, flow at the FCVs, flow at the regasification plants, international connections and others variables. Let us point out that all these variables are also known, in the mathematical systems theory, as control variables.

Once the optimization goal has been chosen, it is important to apply a set of constraints which reproduce the real conditions of the problem. These conditions can be put into three different groups:

- **Physics**: mass conservation equations (1) and pressure loss equations (2), (3), (4) and (5). Notice that this group of conditions coincides with the state equation (7).
- **Security of supply**: imposed mass flow rate at exit points, minimum and maximum pressure allowed at each node of the network and capacity bounds at each pipe of the network.

- **Compressor stations**: every compressor must work accordingly to its operating diagram and its technical characteristics. Figure 1 shows an example of the operating diagram of a compressor. As we can see, the functions defining the diagram imply nonlinear constraints. These constraints are based on works [8] and [7].

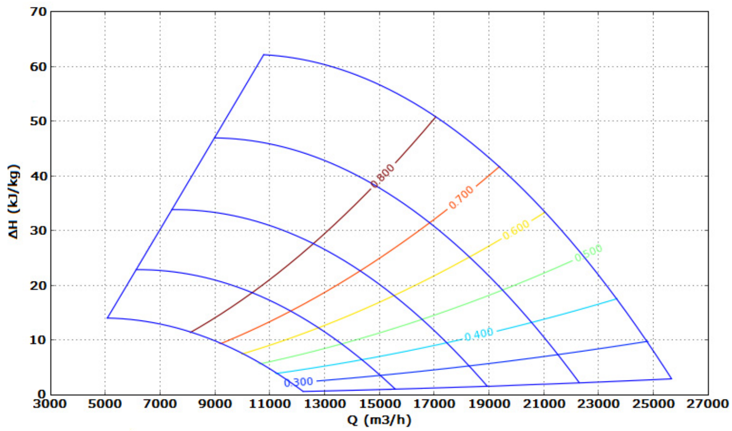

Fig. 1. Operating diagram of a compressor station, where the x-axis represents the volumetric flow rate [m3/h] and the y-axis represents the isentropic head [kJ/kg]. Furthermore, this diagram includes the speed curves and the efficiencies

Lastly, a two-stage procedure to tackle this problem has been developed. In the first stage the complexity of gas physics is reduced while taking all discrete decisions into account. Thus, the first stage provides all discrete decisions and gives an initial solution to the second stage.

The second stage refines the solution obtained by the first stage, providing a result that uses a slightly more precise formulation of the physical constraints and that may be used for the study and management of the gas network.

4.1. First stage

The idea of this stage is to obtain an initial solution, which is used to configure the network (compressor stations, PCVs, ...), disregarding some second order physical effects. Notice that it involves solving a mixed integer nonlinear problem. Indeed, firstly there are a lot of nonlinear aspects, mainly due to the pressure loss in the pipes, the gas consumption and the operation range of the compressors. Secondly, there are binary decisions regarding whether or not a given valve or compressor is active.

This approach does not use the simulator, which means that the group of conditions included in “physics'” (and then in the state equation) are imposed as a set of constraints of the problem. Regarding the optimization goal, the pressures and mass flow rates are independent variables.

One of the algorithms that can be used to get a solution to this problem is the classical Sequential Linear Programming (SLP) algorithm. This algorithm is widely studied in the literature and it has a very good behavior in practice (see [3], chapter 10). It consists in solving iteratively linear approximations of the nonlinear problem until the algorithm finds an optimal suitable solution. There are different ways to linearize the functions and the constraints, e.g. Taylor approximations. Its main characteristics are that it does local search based on bounded size steps at each iteration and it provides a sequence whose limit points are KKT (Karush-Kuhn-Tucker) points.

For our purpose, the classical SLP has one limitation since it does not accommodate binary variables. In order to avoid this limitation, a modified version of the algorithm[1] has been developed allowing to introduce the binary variables. Besides, it also allows unbounded size steps, meaning that at every iteration a mixed integer linear

[1] The authors are currently developing a paper about the modified version of the SLP algorithm. For further details, please contact the authors.

optimization problem is solved. On the downside, it is more common to observe convergence problems, such as cycling, in this modified version of classical SLP than in the standard version.

4.2. Second stage

The aim of this stage is to refine the solution given by the first stage to obtain a result which reproduces the physical behavior of the network and fulfills all the original constraints.

This second stage is based on control theory techniques (see [12]). It employs the simulator to implicitly express the state variables (pressures and mass flow rates) in terms of control variables. In this way, the independent variables for the optimization goal are the control variables. It is important to point out that the conditions of the problem related to the group "physics" are already included by means of the simulator. In consequence, the final solution uses slightly more precise formulations of the physical constraints.

Given that the configuration provided by the first stage contains all the discrete decisions made, this approach deals with a continuous nonlinear problem. Again, the classical SLP can be used as an algorithm to solve the problem but, unlike the first stage, the optimization goal and the constraints are locally linearized by using the derivatives respect to the control variables.

Finally, if we compare this second stage with the first one, it spends more computational time.

5. GANESO

All the mathematical algorithms described above are included in a computer program called GANESO™, which stands for GAs NEtwork Simulation and Optimization. The main functionality is to simulate and optimize gas network transportation problems, providing solutions that can be used to understand the different possibilities of management of a gas transport network.

The active elements of the gas network can be controlled by the user, like different kinds of valves (pressure control valves, section valves, etc.), or handled by the program in the first stage of the optimization procedure.

Importantly, different objective functions can be considered during the optimization process, such as the gas consumed by the compressors and/or the boil-off (gas consumed by regasification plants), or the gas evacuated from a given zone.

This software also includes a number of additional optimization functionalities, for example, it can be used to calculate the maximum demand supported by the gas network and also to track the estimated path of each unit flow from a specific entry point to its exit point.

Turning to the computational aspects, all the aforementioned tools are implemented in FORTRAN 2008 with Objected Oriented Programming and vectorization. In order to facilitate the input and the output of the data, GANESO can read and write Microsoft Excel XML files and can also write Google Earth files. Nevertheless, a graphical user interface is provided to facilitate even more the interaction with the scenario. In particular, a specific plug-in was developed for the open source project Quantum GIS. This plug-in executes the kernel with data defined in the graphical user interface, allowing the user to employ on-line cartography services, such as Google Maps and Open Street Map, or off-line cartography.

Figure 2 shows the Spanish gas network represented on the graphical user interface based on Quantum GIS with the kernel running in a small window.

The user can choose the physical model for the friction coefficient (Weymouth or Colebrook), for the compressibility factor (AGA8 or SGERG88), the optimization solver and other minor options. It is also possible to specify the chemical composition of the gas flowing through the network.

A distinctive feature of GANESO is that, if desired, it delivers a final solution in which all compressor stations are working not only in feasible regimes, but also in efficient ones. For each compressor station, depending on its technical characteristics, there is a different operating domain. Thus, the program also looks for the optimal operating point of every compressor. This feature requires the use of additional nonlinear constraints and binary variables in the first stage of the optimization problem. GANESO provides a visualization tool where the user can check the operating point of every compressor in a given solution, along with its domain and the number of active compressors. This functionality is illustrated in Figure 3.

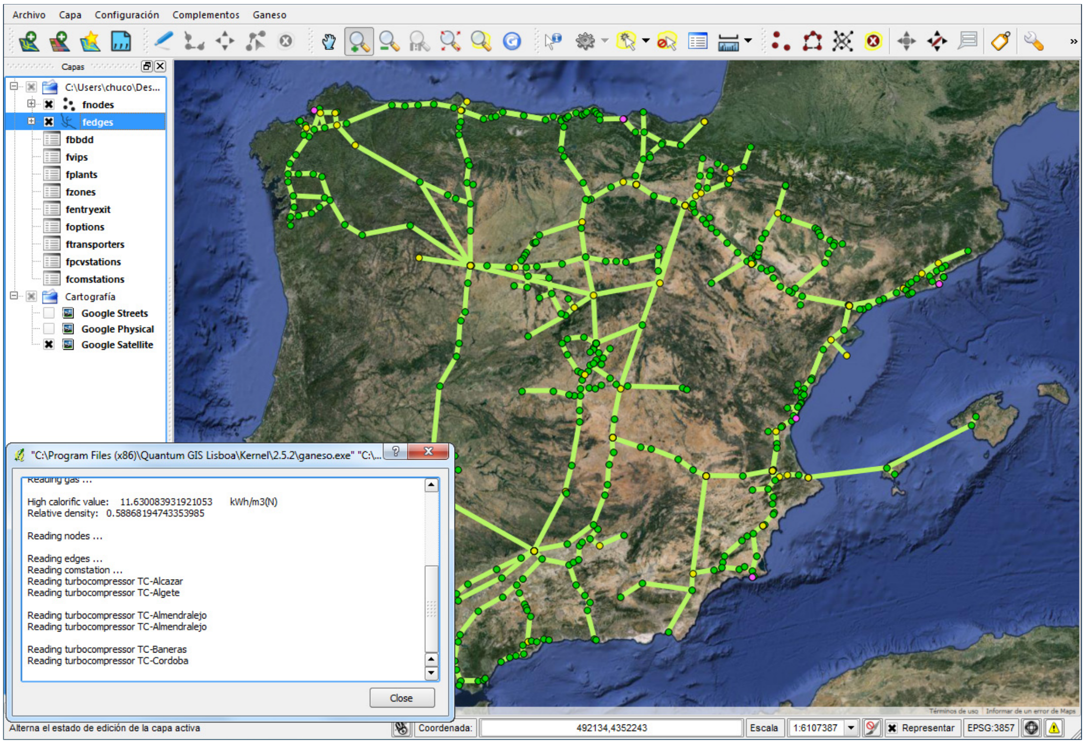

Fig. 2. Spanish gas network on the graphical user interface with kernel running in a small window.

It is worth noting that this program is being developed for Reganosa LNG Company, which is a Spanish Transport System Operator. Since they are continuously using the software for their internal operations and to analyze different operational possibilities of the system, the tool is rapidly improving thanks to the feedback received from them.

6. Application to the Spanish gas network

In this section we are going to simulate and optimize a scenario of the Spanish gas network. In practice, we work with a representation of the Spanish gas network that contains around 500 nodes and 500 edges (over 1000 variables).

The optimization premises that we are going to consider in our scenario of the Spanish gas network are the following:

- International connections and underground facilities are taken as fixed inputs.
- The optimizer has freedom to choose the distribution of flow among the regasification plants.
- The optimizer has freedom to choose how to use compressor stations, PCVs and FCVs.
- The optimization goal is based on the gas consumption in the compressor stations.
- Arrangement of consumptions representing a work day of January with low demand.

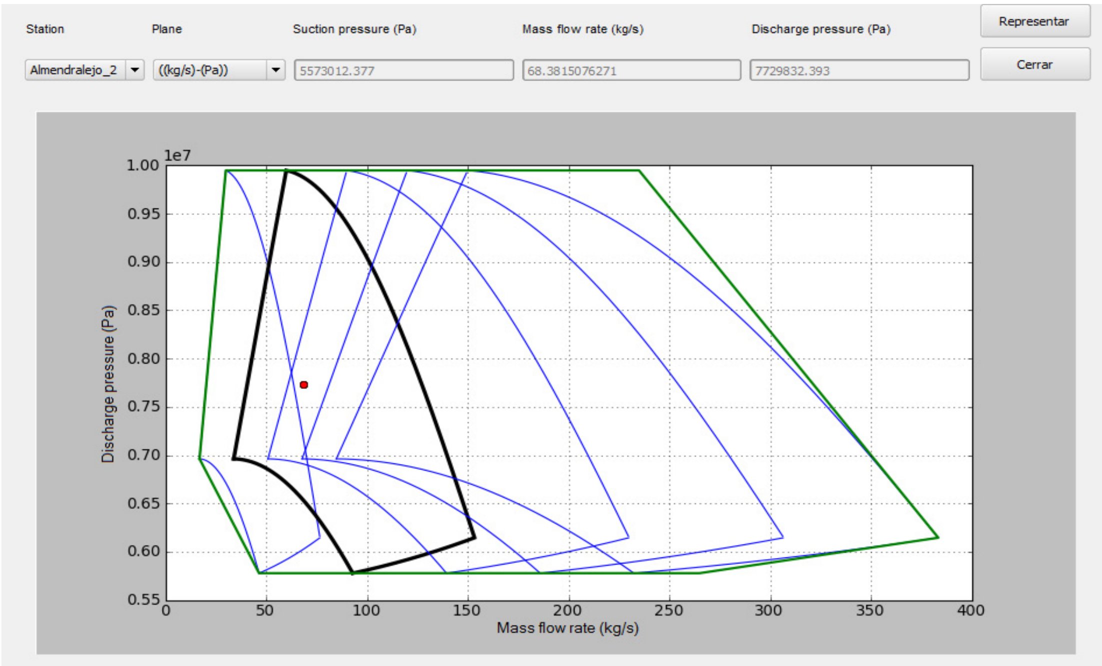

Fig. 3. Tool to visualize the operating point of a compressor station.

Figure 4 shows the solution obtained simulating the scenario of the network with data provided by the Technical System Manager of the Spanish gas network. The compressor stations are labeled with paddles, being the pink ones those which are active. In particular, there are four compressor stations out of eighteen working (Alcazar, Almendralejo, Tivisa and Zamora) and there are no active valves of any kind. It is also possible to see the amount of gas supplied by each entry point of the network. For example, in this scenario, the international connection of Tarifa is the entry point which supplies more gas to the network, about 303 GWh/d. Notice that the use of the elements of the network is a Technical System Manager's decision.

By contrast, in Figure 5 it is possible to see the result of the optimized management. In this case, the software decided to use only one compressor station (Almendralejo), out of eighteen, and to employ a different distribution of the gas flow at the regasification plants. Besides, it is worth noting that the self-consumption cost of the compressor stations decreases, even in Almendralejo.

In Table 1 the gas consumption of every compressor station considering the simulated or optimized management of the gas network is depicted. Based on the last management, the cost could decrease up to 17 % of the usual one.

The flow distribution at the regasification plants is represented in Table 2. Depending on how the network is managed, this distribution can be very different. In the optimized management of this scenario, the total amount of gas supplied from the Southern regasification plants (Huelva and Cartagena) is lower than in the usual management, whereas the Northern ones (Reganosa, Barcelona and Bilbao) introduce more gas in the network.

The computational time of this example was around five minutes in a desktop computer. It is important to point out that Reganosa LNG uses extensively this software on its daily basis with real-life examples. Thus, within this successful running time, the software is able to provide a solution which meets the accuracy requirements of the company.

7. Conclusions

New algorithms to simulate and optimize gas transport network problems have been developed. The optimization algorithm is based on a two-stage procedure. In the first stage, based on mathematical programming techniques, an initial solution which provides a configuration of the gas network is obtained. Then, this initial solution is refined using a second algorithm based on control theory techniques.

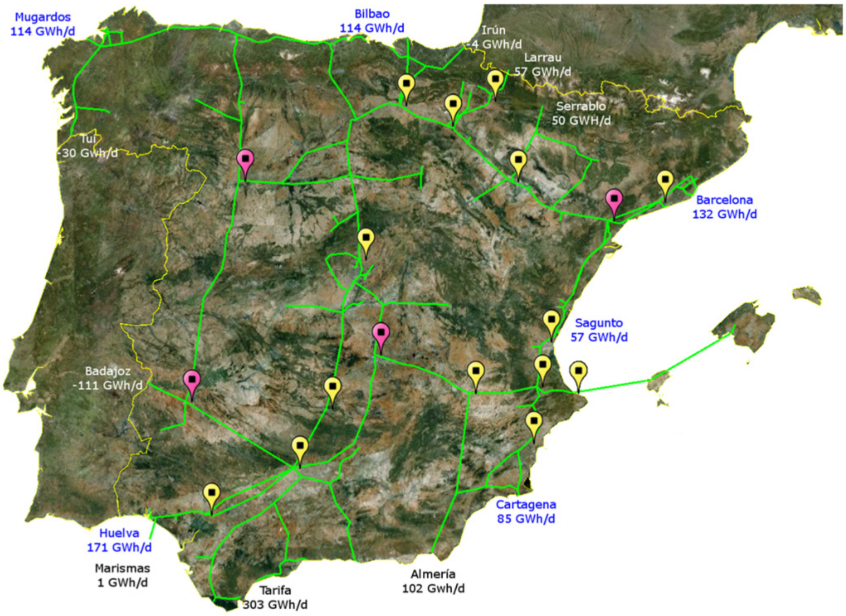

Fig. 4.Simulation of the Spanish gas transport network with data provided by Technical System Manager.

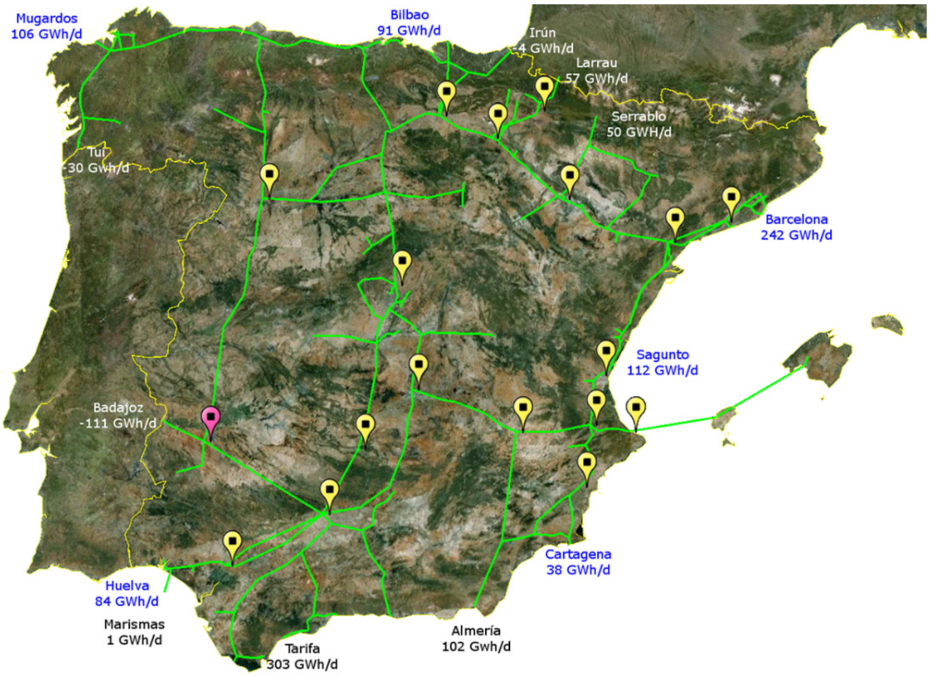

Fig.5. Optimized management of the Spanish gas transport network.

Table 1. Gas consumption in the active compressor stations.

Compressor station	Simulation [GWh/d]	Optimization [GWh/d]
Alcazar	0.2909	-
Almendralejo	0.2650	0.1587
Tvisa	0.2229	-
Zamora	0.1516	-
TOTAL	0.9304	0.1587

Table 2. Distribution of flows at the regasification plants.

Regasification plant	Simulation [GWh/d]	Optimization [GWh/d]
Barcelona	131.8407	241.6383
Bilbao	113.8560	90.6997
Cartagena	85.3920	38.2229
Huelva	170.7840	83.6432
Reganosa	114.1571	106.4408
Sagunto	56.9280	112.3129

Furthermore, the above algorithms have been implemented in a software called GANESO, along with a graphical user interface. It has provided satisfactory results and has proved to be useful for the company which funded this research, Reganosa LNG Company. An application to the Spanish gas network is also presented and analyzed, where the self-consumption cost of the compressor stations could be significantly reduced with an optimized management of the network.

Acknowledgements

The authors would like to acknowledge the support of Reganosa LNG Company and also Ministerio de Economía y Competitividad (Spain) under research project MTM2008-02483 and Xunta de Galicia (Spanish region) under research project EM2012/111.

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